UW Math Circle - Homework 10

In class we talked about a new approach to solving problems: **induction**. When you are using induction, start by proving the problem for a simple **base case**. Then, *suppose* that you can solve the problem for some n and show that you can then solve the problem for $n+1$ (using the fact that you know how to solve the problem for n).

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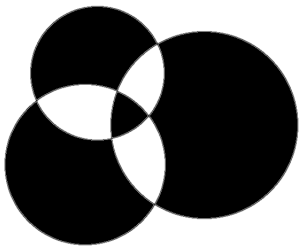
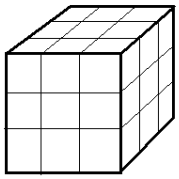
1. Prove that you can cut a square into n squares for any $n \geq 6$.

2. José is cutting paper into pieces. He starts with a sheet of paper and cuts it into either 4 or 6 pieces. He then chooses one of these pieces and cuts it into 4 or 6 smaller pieces. After some time, what are the possible numbers of pieces that José can end up with?

3. An infinite sheet of paper has n circles drawn on it such that no three circles intersect in the same point. Prove that you can color the areas made by the circles in black and white so that no two adjacent areas are of the same color. Now prove the same claim but allowing n circles and m lines such that no three lines or circles intsect in one point.





4. Is it possible to build a $3 \times 3 \times 3$ block with the center $1 \times 1 \times 1$ block cut out using $1 \times 1 \times 2$ bricks?