UW Math Circle - Homework 1

1. In class we showed that n^2 can only have remainder 0 or 1 when divided by 4. What remainders can $n^3 + 2n$ have when dividing by 3?

2.Bulbasaur needs 100 leaves to use razor leaf. He currently has three leaves. He can use vine whip on a leaf to cut it into four smaller leaf pieces, which he can then grow back to full leaf size. Using this technique, can he ever get exactly 100 leaves?

3. Find the remainder which

a) the number $1989 \cdot 1990 \cdot 1991 + 1992^3$ gives when divided by 7.

b) the number 9^{100} gives when divided by 8.

No calculators allowed!

4. José writes the numbers 1 through 99 on some cards. He then shuffles the cards, looks at the first card, and writes down the number on the card minus 1. He then pulls out the second card and writes down the number on the card minus two. He continues on like this until he has 99 numbers written down, and then José multiplies them all by each other. Prove that José will always get an even number.

5. Find all integers n such that $n + 10 | n^3 + 100$.