

UW Math Circle

February 7, 2013

1. Let x and y be integers. Show that $2x + 4y$ is divisible by 13 if and only if $7x + y$ is divisible by 13.
2. Brave Sir Cosmo claims that he can find four consecutive positive integers, a , b , c , and d whose product is a perfect square. Is he correct in his claim?
3. Freddy claims that $n^2 + n + 41$ is a prime number for all positive integers n . Is this true?
4. Is it possible to solve the following puzzle if A , B , C , D , E , and F are all different digits between 0 and 9?

$$AB \times CD = EEFF.$$

- 5.* If a and b are odd integers and n is a nonnegative integer, show that $a^3 - b^3$ is divisible by 2^n if and only if $a - b$ is divisible by 2^n .