

# UW Math Circle

March 7, 2013

1. Is it possible to rearrange the numbers 1, 2, 3, 4, 5, 6 into a 6-digit number that is divisible by 11?
2. Is it possible to form two numbers using only the digits 2, 3, 4, 9 (for example 233439 and 4242) such that one of the numbers is 19 times the other?
3. The number  $8^N$  is written on the whiteboard. Now compute the sum of its digits. Then compute the sum of the digits of that number. Then compute the sum of the digits of that number. Continue this process until you end up with a single number. What is that number if you start with  $N = 2013$ ?
4. Brave Sir Cosmo has some strange copy machines in his office at the Kazakhuzbeki-tajiki embassy. When Cosmo copies a piece of paper with numbers  $A$  and  $B$  written on it:
  - the first copy machine prints a piece of paper with the numbers  $A + 1$  and  $B + 1$  written on it,
  - if  $A$  and  $B$  are even, the second copy machine prints a piece of paper with the numbers  $\frac{A}{2}$  and  $\frac{B}{2}$  on it.

The third copy machine copies two pieces of paper at once – if the first sheet has numbers  $A$  and  $B$  written on it, and the second has numbers  $B$  and  $C$  written on it, the machine prints returns a single piece of paper with the numbers  $A$  and  $C$ .

Question: Is it possible for Brave Sir Cosmo to start with a piece of paper with the numbers 5 and 19 written on it and eventually make a piece of paper with the numbers 1 and 2013?