

UW Math Circle  
Problems – 30 January 2013

1. What are the last two digits of  $99^{2013} + 101^{2013}$  and  $98^{2013} + 102^{2013}$ ?
  
2. There are seven numbers, and the sum of any six of them is divisible by 5. Show that all seven numbers are divisible by 5.
  
3. (a) There are 25 purple, 30 gray, and 35 yellow aliens on a planet. One morning, the aliens go outside to play. When two aliens of different colors meet, they fuse together into a new alien of the third color. (For example, if a purple alien meets a yellow one, they will fuse into a gray one.) After some time, just one alien was left. What color could it have been?  
(b) There are 24 green, 25 blue, and 26 red aliens on another planet. One morning, the aliens go outside to play. When two aliens of different colors meet, they both change their colors to the third color. (For example, if a blue alien meets a red one, they will turn green.) Is it possible that all aliens would eventually be the same color?
  
4. Prove that if  $111 \dots 111$  ( $n$  ones) is a prime number, then  $n$  is a prime number.  
(Hint: Equivalently, show that if  $n$  is not prime, then  $111 \dots 111$  is not prime.)