

UW Math Circle

Problems – 23 January 2013

1. Prove that a whole number is divisible by 4 if and only if the number formed by its last two digits is divisible by 4. (For example, 148 is divisible by 4 because 48 is; 1542 is not because 42 is not.)
2. A triple of positive integers a, b, c is called a Pythagorean triple if $a^2 + b^2 = c^2$, that is, a, b , and c could be the side lengths of a right triangle.
 - (a) Show that in a Pythagorean triple at least one of the numbers must be divisible by 3. (*Hint: Make a table of the remainders of a and a^2 when divided by 3.*)
 - (b) Show that in a Pythagorean triple at least one of the numbers must be divisible by 5.
3. What is the last digit of 777^{777} ?
4. Brave Knight Cosmo is saving the world from an invasion of 150 aliens that have landed in a flying saucer. His force field generator can kill exactly 3 aliens, but 11 new ones spawn in the saucer because of the force field. He can also use a plasma grenade, which can kill exactly 12 aliens and will not cause spawning. Can Sir Cosmo ever kill all the aliens if he has an infinite supply of weapons?

Keep in mind that Cosmo cannot use the weapons if there are not enough aliens. For example, if he tries to use the plasma grenade when there are less than 12 aliens to kill, then Cosmo himself will explode and die, which is undesirable.