UW Math Circle, Spring 2013 - Homework 5

Due May 16, 2013

This week we had a fun lecture on the **Hilbert Hotel**. You can find the worksheet we did in class on the UW Math Circle website.

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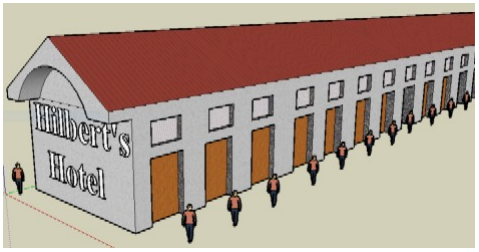
1. This problem is about Hilbert's Hotel:

a) Hilbert's Hotel is compeltely full, but Hilbert's three best friends have just come to town and he needs to find a place for them to stay - can you make room for Hilbert's friends in Hilbert's Hotel?

b) Now, each of Hilbert's friends invites over infinitely many of their friends - how can Hilbert find rooms for each of his friend's friends?

c) Hilbert is planning to expand his hotel by adding five new floors (and each floor will have infinitely many rooms). Will this increase the capacity of his hotel?

d) Now Hilbert wants to add infinitely many floors - will *this* increase the capacity of the hotel?



2. For what nonnegative n is $n! > 2^n$? Prove your answer using induction! (Note: $n! = n(n-1)(n-2)\dots(3)(2)(1)$)

3. Prove that in any group of 2013 people you can always find two who know the same number of people in the group. (Note: every two people either know each other or they do not).