UW Math Circle, Spring 2013 - Homework 3

Due May 2, 2013

This week we learned how to **calculate sums** using induction. Often in math we need to add many numbers together that follow a specific pattern (see below for examples). If we have a hunch as to what the sum could be, we can prove that our hunch is correct using induction. The procedure is the same as it was before: start with a **base case** (where you show that your hunch is correct for some small n) and then prove the **inductive step**. Some of the problems below are very easy - use these as practice for explicitely writing out the base case and inductive step!

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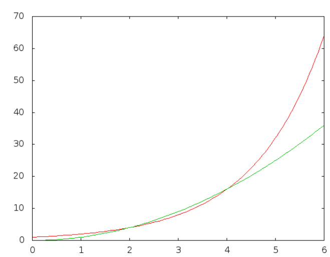
1. Prove that for all integers $n > 1$,

$$\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} > \frac{13}{24}$$

2. Find all non-negative integers n for which $2^n \leq n^2$



3. Prove that for all positive integers n,

$$\frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot 2n} \leq \frac{1}{\sqrt{2n+1}}$$

4. What is the maximal number of 1×4 tiles that will fit in a 6×6 square? Note that the 1×4 tiles cannot overlap or go out of the boundaries of the 6×6 square.