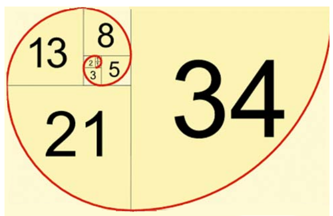
UW Math Circle, Spring 2013 - Homework 2

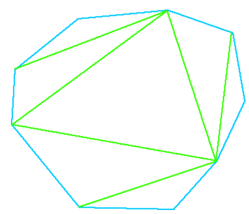
Due April 25, 2013

This week we learned about **strong induction**. Strong induction is very similar to induction: start with a **base case**, then prove the **inductive step**. The difference is that your inductive hypothesis should say "suppose the problem statement is true for *all* k *less than some* n." Notice that this is a stronger statement than regular induction, where the hypothesis was "suppose the problem statement is true for *some* n."

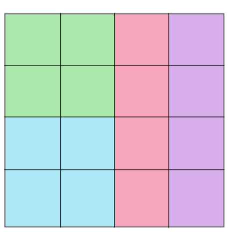


1. The Fibonacci numbers start with $F_1 = F_2 = 1$ and continue with $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. The first few elements of the sequence are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ... Prove that any positive integer can be written as a sum of several Fibonacci numbers.

2. Several diagonals are drawn in a convex polygon so that no two diagonals intersect (but some vertices can have more than one diagonal coming out of them). Prove that you can find at least two vertices of the polygon such that no diagonals start or end at them.



3. Prove that the n^{th} Fibonacci number is divisible by 3 if and only if $4|n$.



4. Steve has a box with 2×2 and 1×4 tiles, such that the bottom of the box is perfectly tiled. All the tiles were dumped out of the box, and in the process one 2×2 was lost. Prove that if Steve replaces this 2×2 tile with a 1×4 tile, it will be impossible for him to tile the bottom of his box again.