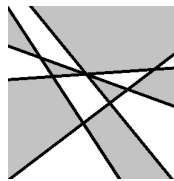


# Mathematical Battle

UW Math Circle

25 April, 2013

1. Brave Sir Cosmo draws some lines in the plane. Prove that he can color the resulting regions black and white so that no two neighboring regions receive the same color. (Two regions are neighbors if they meet along a portion of one of the lines.)



2. Find a simple expression for

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n!.$$

3. Scientists wanted to send a rover around the equator of Mars. They were going to leave fuel at equal intervals along the route, and the rover would refuel as it went. But there was a computer error, and the ship dropped the fuel packages at random places and in random quantities around the equator! All we know is that the total amount of fuel that was dropped is exactly enough for the rover to go around the equator. Prove that the scientists can find a starting place for the rover so it can go around without running out of fuel.

4. Simplify

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{2012 \cdot 2013}.$$

5. Steve wrote a long expression on the board:

$$(1 - \_) \cdot (2 - \_) \cdot (3 - \_) \cdots (99 - \_).$$

He then asked his best student, Alice, to fill in the 99 blanks with the numbers from 1 to 99, using each number exactly once. Finally, he told a particularly annoying student, Bob, to evaluate this expression while everyone else was at recess.

When Steve came back to the room, Bob announced, "I'm too lazy to do it all, but I know the answer will be an odd number!" Could Bob be telling the truth?

6. The nine numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 were split into three groups. Prove that the product of the numbers in at least one of the groups is greater than 71.

7. Scientists have discovered a new genus, the Chess Crocodiles. They live on an infinite chessboard and love to hop around. The Chess Crocodile of species  $n$  moves as follows: it first jumps 1 square in any direction, then turns either left or right and jumps  $n$  squares in that direction. For example, the Chess Crocodile of species 2 is simply a chess knight.

Which of the Chess Crocodile species (for  $n = 1, 2, 3, 4, \dots$ ) can jump to any square from any other square in several moves?