

UW Math Circle

May 2, 2013

Homework

1. If a taxicab can only drive north and east, in how many ways can the cab drive:

- (a) 1 block north and 2 blocks east?
- (b) 1 block north and 3 blocks east?
- (c) 1 block north and 4 blocks east?
- (d) 1 block north and n blocks east?
- (e) 2 blocks north and 2 blocks east?
- (f) 2 blocks north and 3 blocks east?
- (g) 2 blocks north and 4 blocks east?
- (h) 2 blocks north and n blocks east?
- (i) m blocks north and 1 block east?
- (j) m blocks north and 2 blocks east?
- (k) m blocks north and 3 blocks east?
- (l) m blocks north and n blocks east? Can you answer this question without using any algebra?

2. Prove the following identities without using algebraic manipulations.

- (a) $\binom{n}{5} = \binom{n}{n-5}$
- (b) $\binom{n}{5} = \binom{n-1}{5} + \binom{n-1}{4}$
- (c) $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$
- (d) $3\binom{n}{3} = n\binom{n-1}{2}$
- (e) $k\binom{n}{k} = n\binom{n-1}{k-1}$
- (f) $\binom{8}{3} = \binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \binom{5}{2} + \binom{6}{2} + \binom{7}{2}$
- (g) $\binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \cdots + \binom{n}{2} = \binom{n+1}{3}$
(How does this compare to $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$?)
- (h) What do you think $\binom{k}{k} + \binom{k+1}{k} + \binom{k+2}{k} + \cdots + \binom{n}{k}$ should equal?