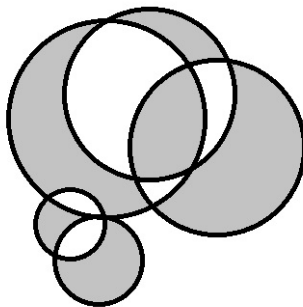


UW Math Circle

April 25, 2013

1. Brave Sir Cosmo drew some circles in the plane. Prove that he can color the resulting regions in two colors so that no two neighboring regions receive the same color. (Two regions are neighbors if they meet along a portion of one of the circles.)



2. (a) What is the sum of the first n odd numbers $(1 + 3 + 5 + \cdots + (2n - 1))$? Make a guess and prove it.
(b) Can you find a geometric argument for your guess?

3. For which n is it possible to write the numbers from 1 to n in a line so that the average of any 2 or more consecutive numbers is never a whole number?

For example, if $n = 4$, we could write 3 4 1 2. None of the averages of 2, 3, or 4 consecutive numbers are whole numbers. But, if $n = 5$, there is no way to do this.