

UW Math Circle

April 18, 2013

1. Brave Sir Cosmo draws some lines in the plane. Prove that he can color the resulting regions black and white so that no two neighboring regions receive the same color. [Here, two regions are neighbors if they meet along a portion of one of the lines.]

2. Find a simple expression for

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n!.$$

3. Scientists wanted to send a rover around the equator of Mars. They were going to leave fuel at equal intervals along the route, and the rover would refuel as it went. But there was a computer error, and the ship dropped the fuel packages at random places and in random quantities around the equator! All we know is that the total amount of fuel that was dropped is exactly enough for the rover to go around the equator. Prove that the scientists can find a starting place for the rover so it can go around without running out of fuel.

4. Simplify

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{2012 \cdot 2013}.$$