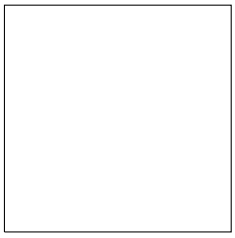
UW Math Circle

Homework: April 4, 2013

1. Show that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9 for any positive integer n.

2. Draw a square S.



   (a) Show that it is possible to dissect S into seven smaller squares.

   (b) Show that it is possible to dissect S into eight smaller squares.

   (c) Show that it is possible to dissect S into nine smaller squares.

   (d) Show that for any $n \geq 7$, it is possible to dissect S into n smaller squares.

3. Suppose you draw N straight lines in the plane so that

   - No pair of lines is parallel (meaning every pair of lines intersects), but
   - any three lines do not intersect.

   Into how many regions does this divide the plane?