UW Math Circle
November 29, 2012

1. Brave Sir Cosmo's space ship has 10 stairs from the door to the ground. In how many different ways can he get from his ship to the ground if he can...

   (a) leap from his ship to one of the stairs and then from that stair to the ground?

   (b) step on only two stairs before he reaches the ground?

   (c) visit any number of stairs before reaching the ground?

2. The Uzbekikazakhturkistani alphabet has five letters: a, r, k, y, and z. How many words are there in the Uzbekikazakhturkistani language if...

   (a) a word is a sequence of 4 letters.

   (b) a word is a sequence of at most 4 letters.

   (c) a word is a sequence of 4 letters and no letter is repeated.

3. 10 lines are drawn in the plane. Any two of the lines intersect, but no three of them intersect. How many points of intersection are there? Into how many regions do the lines divide the plane?

4. How many four digit numbers use the numbers 1, 2, 3, and 4? What is the sum of all of these numbers?

5. Ten swimmers compete in a race.

   (a) In how many different orders can the swimmers finish the race?

   (b) The gold, silver, and bronze medalists get to stand on a podium at the end of the race. In how many different ways can the medals be awarded to the swimmers?

   (c) Two of the swimmers, Brave Sir Cosmo and his brother, Hank, are representing Uzbekikazakhturkistan. If all swimmers are equally likely to win the race, what is the probability that both Cosmo and Hank win medals?

6. 10 numbers are written in a row. The sum of any three consecutive numbers is 7, and all 10 numbers add up to 20. What is the 7th number in the row?