UW Math Circle

October 25, 2012

The Pigeonhole Principle. If $n+1$ pigeons are placed into n holes, then there will be at least one hole with more than one pigeon in it.

Proof: Let's pretend we can put $n+1$ pigeons in n holes so that all the holes have no more than one pigeon. Then there would be no more than n pigeons in total. But we have $n+1$ pigeons! That's a contradiction. So, our assumption is false. At least one of the holes has more than one pigeon. $\square$

This simple result is also known as **Dirichlet's Principle**. There is a more interesting version:

The Generalized Pigeonhole Principle. If $kn+1$ pigeons are placed into n holes, then there will be at least one hole with more than k pigeons in it.

Proof: Let's pretend we can put $kn+1$ pigeons in n holes so that all the holes have k pigeons or fewer. Then there would be no more than kn pigeons in total. But we have $kn+1$ pigeons! So, there must be at least one hole with more than k pigeons. $\square$

Problems:

1. There are 22 Math Circlers in the class. Is it true that we can find four of them who were born on the same day of the week?

2. The Brave Knight Cosmo is baking a huge pie to celebrate his triumphant victory over the monster that had been terrorizing Uzbekikazakhturkistan (by decree of the Uzbekikazakhturkmeni government, he is not legally allowed to tell you how he defeated the monster). He rolls out the dough into a 4-foot-by-4-foot square, then leaves to buy some sugar at the store. When he comes back, he sees that 33 flies have landed on the dough and are feeding on it! Angry, Cosmo reaches for his 1-foot-by-1-foot square baking pan. Show that he can bring the baking pan down on the dough to kill at least 3 flies with one blow.

3. What is the greatest number of bishops you can place on an 8×8 chessboard so that no two of them attack each other?

4. Can you fill a 3×3 table with the numbers 0, 1, and 2 in such a way that the sums along the rows, columns, and diagonals are all different? (For example, in the table below, there are several sum that are 3 and several that are 4, so that configuration doesn't work.)

			╱**4**
1	0	2	**3**
2	2	1	**5**
0	1	1	**2**
3	**3**	**4**	╲**4**

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Homework – Week 4

1. No fewer than 5,000,000 people are in Seattle for the Northwest Grunge Rock Festival. Nobody has more than 1,000,000 hairs on his or her head. Prove that there are two people at the festival with the same number of hairs on their heads.

2. 20 out-of-state bands are participating in the Grunge Rock Festival. Each band consists of 11 members. 200 players arrived by plane, but then there was a snowstorm and the airport had to be closed. One person drove in. Show that at least one band now has all of its members present.

3. Out of any 1001 integers, can I always find two whose difference is divisible by 1000?

4. (a) Donald is on a business trip to the planet Circia. He has a new space suit with 10 pockets, and he wants to put his 44 business cards into them. Could he do it so that each pocket contains a different number of cards?

   (b) 10 people are at a party. Some of them know each other, others do not. Could they each have a different number of friends among the people at the party?

5. 64 whole numbers were written on the squares of a chessboard. It turned out that the sums of the numbers in all the rows and all the columns were even. Show that the sum of all the numbers on the black squares is also even. *(Hint: Try coloring the board in four colors.)*