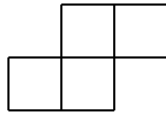# UW Math Circle
October 18, 2012

1. Steve has a tropical hybrid fruit tree in his back yard that has 25 starfruits and 30 papayas on its branches. Every morning, he picks two fruits from the tree. If he picks two of the same type of fruit, a new papaya grows that day. If he picks a papaya and a starfruit, a starfruit grows that day. After a few months, there is only one piece of fruit left on the tree. What fruit could it have been?

2. The donkey is a new type of chess piece that can move either one square left/right and 4 squares up/down OR 3 squares left/right and one square up/down. There are two donkeys on an 8×8 chessboard – one in the upper left corner and another in the lower right corner. Every second, they both make a move. Is it possible that they could ever run into one another?

3. For which numbers m and n is it possible to cover an $m \times n$ chessboard with non-overlapping 4×1 dominoes? Prove that your answer is correct by showing that a covering is possible when you say it is possible and showing that it is impossible in all other cases.

4. Kolya goes to the market to buy some fruit. Pears cost 89 cents, apples cost 59 cents, and bananas cost 29 cents. Kolya buys 9 pieces of fruit, and the cashier charges him $5.16. Kolya immediately knows that the cashier made a mistake. How does he know? (Assume there is no tax.)

5. A 100×100 chessboard is cut into many small squares, each of which has an odd number of squares on each side. Show that the number of smaller squares whose center is black is equal to the number of smaller squares whose center is white.

6. A **tetromino** is a domino that looks like this:

   What is the maximum number of tetrominoes that can be placed on an 8×8 chessboard? (Tetrominoes can be rotated or flipped and placed on the board.)

UW Math Circle
Homework – Week 3

1. Harry goes to the store to buy some candy bars. He buys 24 kit-kat bars, 17 snickers bars, and 16 peanut butter cups. Snickers bars and peanut butter cups cost the same price. In the end, the cashier charges Harry $18.65, and Harry immediately knows that the cashier made a mistake. How does he know? (Assume that each candy bar costs a whole number of cents and that there is no tax.)

2. One day, while the Man in the Yellow Hat is out for a walk, Curious George tears 15 pages out of a book that he finds on the bookshelf. Is it possible for him to add the numbers of all the pages he tears out and get 1858?

3. What is the last digit of the sum

$$1^2 + 2^2 + \cdots + 99^2$$