Math Circle - Homework 8

*Recall from class that the expression $\binom{n}{k}$ represents the number of ways of choosing k objects from a collection of n total objects. For this assignment, you can and **should** leave your answers in this notation, whenever it makes sense to do so.*

1. (a) In how many ways can you place five *X*s and four *O*s on a Tic-Tac-Toe board?
(b) What is the total number of boards from part (a) in which there is at least one row, column, or diagonal filled with three *O*s?

2. Chris is going on vacation and has 5 different books that he has not yet read. If he brings any number of the books with him (anywhere from all 5 to none of them), how many possibilities are there for the different collections of books he can bring?

3. 10 men named Jon, 8 named John, and 7 named Johan entered a race. In how many distinguishable ways can these 25 men finish the race? Two finishes are considered indistinguishable if the order of names is the same in the final ranking.

4. There are 7 delicious pies cooling on a windowsill, accompanied by 4 ravens closely eyeing them. Each pie is a different flavor. How many different combinations of pies will remain if each raven eats either one or zero pies?