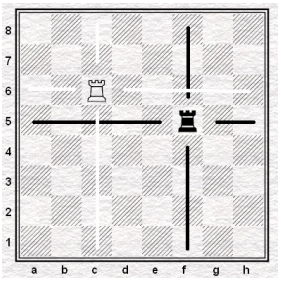
UW Math Circle - Homework 7

Recall from class that the expression

$$\binom{n}{k}$$

represents the number of ways to choose k balls from n balls. For this homework you can (and should!) leave your answers in this notation. It doesn't mean anything to us when you give us a number such as 210, but writing $\binom{10}{4}$ gives us a much better understanding of where you answer is coming from.

1. Chris has 6 friends. Every evening he invites three friends over for dinner, but he never wants to invite the same group over twice. For how many evenings can Chris continue this?



2. How many ways are there to put a black rook and a white rook on a chessboard so that they do not attack each other? How many ways are there to put a black king and a white king on a chessboard so that they do not attack each other?

3. Prove that

$$1 + 2 + 3 + \cdots + n = \binom{n+1}{2}$$

A problem that we solved in class might be very useful here! The problem involved cities that were connected by air routes...

4. Alex made up a new language called "Alexian". His language uses all 26 letters of the alphabet, and a word is any combination of *six* letters that has exactly one A. How many Alexian words are there? *Challange problem:* generalize this to show that

$$26^6 = \binom{6}{6} + \binom{6}{5} \cdot 25 + \binom{6}{4} \cdot 25^2 + \binom{6}{3} \cdot 25^3 + \binom{6}{2} \cdot 25^4 + \binom{6}{1} \cdot 25^5 + \binom{6}{0} \cdot 25^6$$

Aksjex yialer lyxhia!