# Math Circle - Homework 6

1. Suppose you have five points with integer coordinates on a 2D plane. Show that exists a pair of these points such that the midpoint between them also has integer coordinates.

2. Alex had an election night party and invited six of his friends. Among his friends, some knew each other, and some were total strangers to each other. Prove that among the six guests, there were at least 3 who all knew each other, or 3 who were all strangers.

3. Chris has a circular disc of cork board and wants to make a simple dartboard, so he splits the disc into 20 equal sectors. On a dartboard, each sector counts as some integer number of points between 1 and 20. Chris cannot remember which number goes where, so he randomly assigns a different integer $1-20$ to each of the 20 sectors. Is it possible for Chris to arrange the numbers in such a way that there are no 3 consecutive sectors whose values sum to 32 points or more? *Hint. You may want to use the fact that* $1 + 2 + 3 + \cdots + 20 = \frac{21 \cdot 20}{2}$.
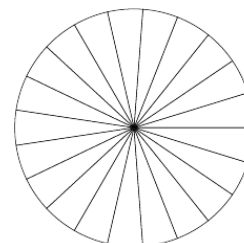
4. Is it possible to construct a solid rectangular $4 \times 3 \times 3$ block from smaller blocks shaped as pictured?