Math Circle - Homework 5

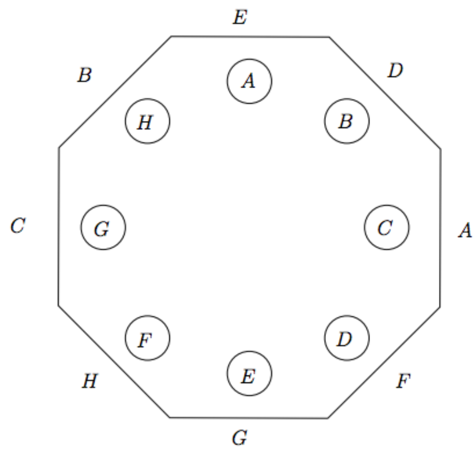
The Pigeonhole Principle. *If we have n pigeonholes and at least $n + 1$ pigeons are placed in them, then at least one pigeonhole will contain* more than one *pigeon.*

1. Every day, the three friends Luke, Han, and Chewbacca line up for some ice cream outside of Mos Eisley Cantina. Prove that during each week, there will be two days in which they line up in the exact same order. A week is still seven days in Mos Eisley.

2. Every point on the plane is colored as either black or white. Show that there must be at least two points of the same color at a distance 1 away from each other. *Very challenging extra problem – only do after finishing everything else: Show the same result, now assuming the plane is colored in three colors.*

3. Eight friends sit down around an octagonal game table for a poker tournament, one player at each edge. Each place at the table had been marked with the name of one of the players, but *none of them* sit at their correct spot. Prove that it is possible to rotate the table so that at least two of the friends are sitting in the correct places.

In the example, the players are A, B, C, D, E, F, G, H. Places are marked with player names in circles, and the players are sitting on the outside of the table. Here, if the table is rotated two spots counterclockwise, then B, D, G, F, H are all at their correct spots.



Source: Mark Flanagan, University College Dublin

4. Choose any 20 distinct numbers from the set of integers

$$\{1, 4, 7, 10, 13, 16, \ldots, 91, 94, 97, 100\},$$

where each subsequent number in the set above increases by 3. Prove that no matter what 20 numbers are in your collection, there must exist two integers in your set of 20 integers whose sum is 104. *Hint: How many pairs in the original set sum to* 104*?*

Source: Putnam Exam, 1978