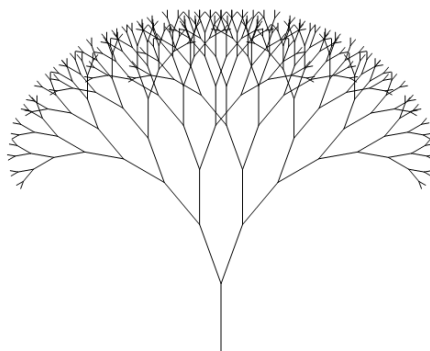


Math Circle - Homework 3

1. A mathematician is wandering through the desert when he comes across a tree with 25 apples and 30 oranges. This tree is magical: when you pick any two fruit of the *same* kind from the tree, another orange grows back; when you pick two *different* kinds of fruit, an apple grows back. The mathematician was very hungry and ate until there was only one fruit left hanging — the final fruit which grew back after taking the tree's last two fruits. Was this final fruit an apple or an orange?



2. There are 7 obscure clocks on the wall of a the Math Department office. Each clock has no hour-hand, and all the minute-hands are pointing straight up. In one move, you are allowed to move the minute hands of any four clocks up by exactly 15 minutes. Is it possible to reach a situation in which all the minute-hands are pointing straight down?



3. Thirty piñata live on Piñata Island. 13 piñata are filled with chocolate, 5 are filled with peppermint, and 12 are filled with butterscotch. Whenever two piñata with different fillings meet, they both completely change out their fillings to the third type. Is it possible that all 30 piñata might eventually all be filled with the same type of candy?

4. Chris is trying to completely tile a 10×10 board with T-shaped tiles so that none of them overlap. Show Chris that, despite his very best efforts, he will never succeed.

