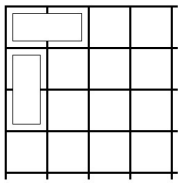
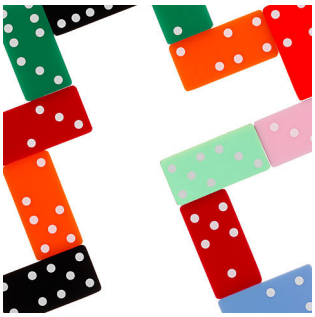
Math Circle - Homework 2

1. Steve is really bored. He decides to spend his time trying to cover a 5×5 square with 1×2 dominoes. Will he ever be successful? Can you generalize your answer (for what kinds of boards will be be or not be successful?)



2. Kolya has 11 whole numbers (some of which are negative). He notices that when he multiplies all of his numbers he gets 10810104834375, but when he adds his numbers he gets 0. Christian (without even using a calculator!) immediately points out that this is impossible - how does he know?



3. Alex is playing dominoes with Chris. At the end of the game, Chris and Alex have a chain of all 28 dominoes, and the number on one end of the chain is 5. What is the number on the other end of the chain?

Clarification: there are 28 *dominoes in a set, and each possible combination of numbers between* 1 *and* 6 *has a domino. When playing dominoes, you must place a domino so that adjacent numbers match. For example, you can place a [3:4] next to a [4:5], but not next to a [5:6].*

4 After Steve got bored of covering a 5×5 square with dominoes and moved on to covering an 8×8 board. Just before Steve started covering the board, Alex cut off two opposite squares from the board. Is it possible for Steve to ever cover the board with dominoes? Can you generalize your answer?

5. Challenge Problem: A grasshopper jumps along a line. His first jump takes him 1cm, his second 2 cm, and so on. Each jump can take him to the right or to the left. Show that after 1985 jumps the grasshopper cannot return to the point at which he started.