

Things to Think on Week 6

1. Warm-up.

(a.) Consider Eulerian paths, Eulerian cycles, Hamiltonian paths, and Hamiltonian cycles. Determine which of these implies another. For example, if a graph has a Eulerian cycle it certainly has a Eulerian path, so you would write:

a graph has a Eulerian cycle \Rightarrow a graph has a Eulerian path

(b.) Show that if A and B are two vertices on a tree, then there is a *unique* path connecting them.

2. We showed in class that *if* a graph has a Eulerian path, *then* it has either 0 or 2 vertices of odd degree. Prove that the opposite is true. Namely, show that if a graph has exactly 2 vertices of odd degree it has a Eulerian path and if it has 0 vertices of odd degree it has a Eulerian cycle.

3. A group of islands are connected by bridges in such a way that one can walk from any island to any other. A tourist walked around every island, crossing each bridge exactly once. He visited the island of Thrice three times. How many bridges are there to Thrice, if

(a) the tourist neither started nor ended on Thrice;

(b) the tourist started on Thrice, but didn't end there;

(c) the tourist started and ended on Thrice?

4. **Definition:** the distance between two vertices on a graph is the length of the shortest path between them. Given three vertices A , B , and C , show that the distance from A to B plus the distance from B to C is less than or equal to the distance from A to C .

5. There are 20 points inside a square. They are connected by non-intersecting segments with each other and with the vertices of the square in such a way that the square is dissected into triangles. How many triangles do we have?

6. There are two nails in a wall. Is it possible to hang a picture frame on them so that if either nail is removed, the frame will fall?