Things to Think on Week 7

1. Is it possible to place a knight on a chess board, and perform a sequence of moves such that every possible move that a knight could make on the board is performed *exactly* once? Is it possible to do this in such a way that the knight ends where it began?

2. Is it possible to place a knight on a chess board, and perform a sequence of moves such that the knight occupies every square on the board *exactly* once? Is it possible to do this in such a way that the knight ends where it began?

Definition. *Given a graph, G, we define a new graph $L(G)$ (called the "line graph" of G) as follows: there is one vertex in $L(G)$ for every edge of G, and two vertices of $L(G)$ are connected by an edge precisely when the corresponding edges in G share a vertex.*

3. Draw the line graph of K_4. Draw the graph corresponding to a "cube" by drawing two squares and connecting corresponding vertices. What is the line graph of this graph? Does it remind you of another shape?

4. Prove that if G has an Eulerian cycle, then $L(G)$ has both an Eulerian cycle and a Hamiltonian cycle.

5. There are two *different* connected graphs with line graph K_3. Find them.

6. Let G be a graph with v vertices and e edges. Suppose the vertices have degrees $d_1, ..., d_v$. How many vertices and how many edges does $L(G)$ have? Draw some examples to get yourself started!