

Things to Think on Week 5

1. The following exercises are meant to be review and should help you get comfortable with graphs
 - (a) Prove that the sum of the degrees of the vertices of any finite graph is even
 - (b) Show that every simple graph has two vertices of the same degree.
 - (c) Prove that if u is a vertex of odd degree in a graph, then there exists a path from u to another vertex v of the graph where v also has odd degree.
 - (d) Show that if n people attend a party and some shake hands with others (but not with themselves), then at the end, there are at least two people who have shaken hands with the same number of people.
2. Show that a tree with n vertices has exactly $n - 1$ edges. Show that any connected graph with exactly $n - 1$ edges and n vertices is a tree.
3. Show that for any *connected*, planar graph (this just means that you can draw the graph on a piece of paper without the lines crossing), the formula $V - E + F = 2$ holds, where V is the number of vertices, E is the number of edges, and F is the number of faces. *Hint, try induction!*
4. A *Hamiltonian Path* is a path that visits every vertex exactly once. Show that any complete graph has a Hamiltonian Path.