Things to Think on Week 3

1. Fred was able to use Mathematical Induction to prove that all horses in the world are brown. He reasoned as follows:

    (a) Base: At least one brown horse exists.

    (b) Induction: Assume *any* collection of k horses are brown. Now take a collection of $k + 1$ horses. If we remove one horse, we have a collection of k horses- so they are all brown. If we remove a different one (from the original $k + 1$), we have a collection of k horses, so they are all brown. But every horse is either in the first collection or the second, so they are all brown.

    Where does his logic go wrong?

2. Suppose you have a convex polygon with several diagonals such that no two intersect. Show that you can always find two vertices such that none of the given diagonals contain either of these points. (Hint: Use strong induction and be careful.)

3. Show that that, for any $n \geq 0$, $n^3 + 2n$ is divisible by 3.

4. There are 8 numbers written on a board, all less than 15. Someone writes down all of the positive differences. Show that at least two of these differences will be the same.