Things to Think on Week 7

1) 175 Humpties cost more than 126 Dumpties. Prove that you cannot buy three Humpties and one Dumpty for one dollar.

2) Problem sets for the upcomming Olympiad for grades 5-10 are compiled so that each set contains eight questions, and there are exactly three questions in each grade which are not used in the other grades' olympiads. What is the maximum possible number of questions used by the problem committee?

3) You are stranded on an island where all of the inhabitants are either knights (who always tell the truth) or knaves (who always lie).

    a)What should you ask a complete stranger (could be a knight or a knave) to find out whether they own a pet crocodile?

    b)One of the islanders, Alan, while standing next to Brittany says "one of us is a knave". Is Alan an knave or a knight? What about Brittany?

4) In a warehouse N containers marked 1 through N are arranged in two piles. A forklift can take several containers from the top of one pile and move them to the top of the other pile. Prove that the containers can be arranged in one pile with increasing numbers with $2N - 1$ such operations of the forklift.

5) You have nine coins, one of which is a counterfeit and is lighter than the others. Using no more than two weighings on a balancing scale determine which coin is the counterfeit.