

Things to Think on Week 2

1. In how many ways can you choose 10 cards from a deck of 52 cards so that (a) there is **exactly** one ace among the chosen cards? (b) there is **at least** one ace among the chosen cards?
2. Ten points are marked on a plane so that no three of them lie on a straight line. How many triangles are there with vertices at these points?
3. Prove that

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n} = 0.$$

4. Prove that one can choose evenly many objects from a collection of n objects in 2^{n-1} ways.
5. Six boxes are labeled 1 through 6. How many ways are there to distribute 20 identical balls between the boxes (some of the boxes are allowed to be empty)?