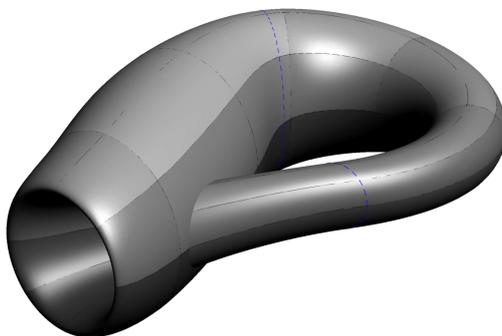


# Math Circle - Spring 2012 - Homework 5

*A mathematician named Klein  
Thought the Möbius strip was divine.  
He said, "If you glue  
The edges of two,  
You'll get a strange bottle like mine."*

**1. (5+10 points)** The **connected sum** of two surfaces  $A$  and  $B$  (written  $A\#B$ ) is defined as follows. Cut out a little circle from each of the surfaces, then glue the resulting parts together at the holes. Convince yourself that the connected sum of two tori,  $T^2\#T^2$ , is a donut with two holes.

- (a) **(5 points)** What is  $S^2\#T^2\#S^2\#T^2\#S^2$ , where  $S^2$  is the surface of a sphere and  $T^2$  is a torus?
- (b) **(10 points)** What is  $P^2\#P^2$ , where  $P^2$  is the projective plane? (*Hint: Look at last week's homework!*)



**2. (10 points)** Take two strips of paper and tape them perpendicularly to make a cross. Now take the first strip and tape the ends into a cylinder. Tape the ends of the other strip with a twist to make a Möbius strip. Finally, cut both strips in half. (You'll have to cut four corners at the intersection.) What do you get? How many sides and edges does it have?

**3. (10 points)** Some Flatlanders living on a Möbius strip decide to send expeditions to the north, south, east, and west. Will they ever meet each other? How can they tell if the strip has 1 twist or 2?