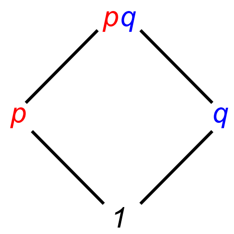
Math Circle - Spring 2012 - Homework 2

1. (10 points) Let S be the set $S = \{1, 2, 3, 4, 5\}$. How many different equivalence relations $\sim$ can you define on S which have exactly two equivalence classes?

2. (10 points) Recall the partial order $\propto$ on the positive integers by *divisibility*. That is, $n \propto k$ if n divides k. For example, $4 \propto 20$ and $5 \propto 20$, but $3 \not\propto 17$.

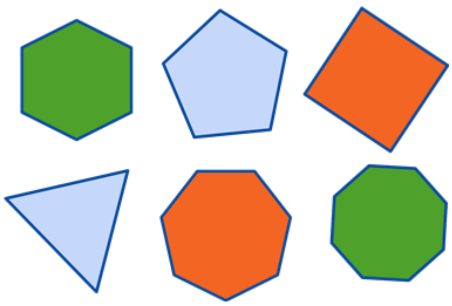
Suppose that p, q, and r are **distinct** prime numbers. Draw the divisibility partial order for (i) pqr and (ii) pq^2.



3. (10 points) Let S be the collection of all 2-dimensional polygons. Define a relation $\sim$ on S by saying that for two shapes X and Y, we have $X \sim Y$ if $\text{area}(X) = \text{area}(Y)$, where area$(X)$ denotes the total area of the figure X.

(a) Prove that $\sim$ is an equivalence relation.

(b) Come up with a way of nicely representing (drawing) the quotient set $S/\sim$.



4. (15 points) Let $S = \{(x, y) : x \text{ and } y \text{ are integers, and } y \neq 0\}$. Define a relation $\sim$ on S by

$$(x, y) \sim (a, b) \qquad \Longleftrightarrow \qquad xb = ya.$$

(a) (5 points) Prove that $\sim$ is an equivalence relation.

(b) (10 points) Describe $S/\sim$ by recognizing it as another mathematical structure that we are much more used to dealing with.