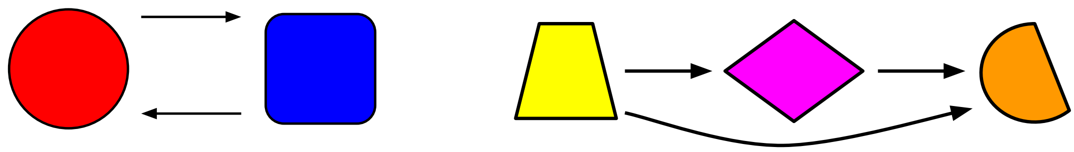
# Math Circle - Spring 2012 - Homework 1

**1. (10 points)** Either prove the following statement or give a counterexample relation on the integers. A relation which is both *symmetric* and *transitive* is also necessarily *reflexive*.



**2. (10 points)** Let $S$ be the set $S = \{1, 2, 3, 4, 5\}$. How many different equivalence relations $\sim$ can you define on $S$ which have exactly two equivalence classes?

**3. (10 points)** Prove that the following is an equivalence relation on the real numbers $\mathbb{R}$ (in other words, show it is symmetric, transitive, and reflexive):

$$x \sim y \quad : \quad x - y \text{ is a rational number.}$$

*Note.* As an example of this relation, $\pi \sim (\pi + 2/5)$ [because $\pi - (\pi + 2/5) = -2/5$ is rational], but $\pi$ is not related to $\sqrt{2}$ [because $\pi - \sqrt{2}$ is not rational].

**4. (10 points)** Recall that the *power set* of $S = \{1, 2, 3\}$ is the collection $\mathscr{P}(S)$ of all subsets of $S$. In particular, $\mathscr{P}(S)$ has 8 elements in it:

$$\emptyset \qquad \{1\} \qquad \{2\} \qquad \{3\} \qquad \{1,2\} \qquad \{1,3\} \qquad \{2,3\} \qquad \{1,2,3\}.$$

There is a natural relation $\propto \; = \; \subseteq$ on these 8 subsets.

**(a)** Which of the five types of relations (symmetric, transitive, reflexive, anti-symmetric, total) does the relation $\subseteq$ satisfy?

**(b)** Come up with a picture which depicts this relation $\subseteq$.