Math Circle - Homework 5

1. (**10 points each**) Finish the problems from the group worksheet on induction.

2. Just like in finding that $1+2+3+\ldots+n=\frac{n(n+1)}{2}$, induction can also be useful in proving algebraic identiies with sums.

   (a) (**10 points**) Evaluate

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{(n-1)n}.$$

   Can you also find a clever way to do it without induction? *(Hint: Try to see what this is for small numbers n, then make a guess about what it is in the general case.)*

   (b) (**10 points**) Show that

$$1+8+27+\ldots+n^3=(1+2+3+\ldots+n)^2.$$

   (Hint: Do we already have a formula for the right side?)

   (c) (**10 points**) **Challenge:** Show *(by induction, of course)* that

$$1+4+9+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}.$$

3. (**10 points**) Suppose we have a very large chessboard, size $2^n\times 2^n$, with the top left corner removed. Is it possible to tile this board with "L" shapes, as shown below?