Think on These Things Week 5

1. Two players, A and B, take turns playing a game. There is a pile of 30 matchsticks. At a turn, the player must take one or two sticks. The player who takes the last stick wins. Who has a winning strategy? What if each player could take one, two, three, or four sticks at each turn? What's the pattern here? What can you say in general about games where there are n sticks and players can take $1, 2, ...,$ or m sticks?

2. You and your friend are going to share a pizza. You open up the box and see that there are 20 pieces, but whoever cut the pizza wasn't very good because the slices were all different sizes. In any event, you flip a coin to see who gets to pick first, and you win! Of course, as everyone knows, it's rude for someone to take one piece, and then someone else to take a piece from the other side: pizza must always look like a PacMan. So you get to take the first piece, but your friend has to take one of the two adjacent pieces, and then you have to either take the piece your friend didn't, or the one adjacent to the piece they just took, etc. Can you ensure that, at the end of the night, you've eaten the most pizza, by volume?

3. The numbers $1, 2, ..., 20$ are written on a board. Matt Damon is allowed to erase any two numbers, a and b, and replace them by $ab + a + b$. After 19 such operations, what are all the possible numbers that could be on the blackboard?

4. There are six sparrows sitting on six trees, one sparrow on each. The trees stand in a row, with 10 meters between any two neighboring trees. If a sparrow flies from one tree to another, then at the same time some other sparrow flies from some tree to another the same distance away, but in the opposite direction. Is it possible for all the sparrows to gather on one tree? What if there are seven trees and seven sparrows?

5. Is it possible for a chess knight to pass through all the squares of a $4 \times N$ board having visited each square exactly once, and return to the initial square?