Think On These Things- Week 3

1. Five cups are lined up on a table. You are allowed to choose two at a time and flip them (from upside-down to rightside-up or vice-versa). If all the cups start facing up, is it possible to perform a sequence of these moves to get them all turned down? What if there are 7 cups?

2. How many bishops can you place on an 8×8 chessboard such that no two are attacking each other? (Recall that bishops are allowed to move diagonally on the board, and two bishops are said to be attacking one another if it is possible to move one to take the other.)

3. Alice and Bob take turns placing dominoes on a 21×21 board.

   If they are working together to cover the board, can they succeed? If so, what should their strategy be? If not, why not?

   What if the center square is cut out?

   Now they are playing against each other: the person who cannot place a domino on the board loses. Who has a winning strategy? What is it?

4. An 8×8 board has all of its squares colored white except the top left corner, which is colored black. You are allowed to reverse the colors in a whole row or a whole column. Is it possible to make all of the squares black?

5. There are some chameleons on an island. At some point there are 13 red chameleons, 15 green chameleons, and 17 blue chameleons. Whenever two chameleons of a different color meet, they both turn into a chameleon of the third color. Is it possible that after some time all of the chameleons become the same color?