THINGS TO THINK ON, WEEK 2

1. Two players, A and B, take turns playing a game. There is a pile of six matchsticks. At a turn the player must take one or two sticks from the remaining pile The player who takes the last stick wins. Player A makes the first move and each player always makes the best possible move. Who wins the game?

2. The numbers $1, 2, ..., 2011$ are placed on the blackboard. The janitor erases two of the numbers and replaces them by their positive difference. Then he does this again- picking two numbers from the new list and replacing them with their positive difference. He does this until only one number remains on the board. Can this number be zero?

3. Three subjects- A, B, and C- were all perfect logicians. Each could instantly deduce the consequences of any set of premises. Also, each was aware that the others were perfect logicians. The three were shown seven stamps : two red ones, two yellow ones, and three green ones. They were then blindfolded, and a stamp was pasted on each of their foreheads; the remaining four stamps were placed in a drawer. When the blindfolds were removed, A was asked, "Do you know one color you definitely do not have?" A replied, "No." Then B was asked the same question and he also replied, "No."

Is it possible, from this information, to deduce the color of A's stamp, or of B's, or of C's?

4. Forty-five points are chosen along a line AB but none are contained in the line segment AB. Prove that the sum of the distances of the points to A cannot equal the sum of the distances of the points to B.

5. A snail crawls along a plane with constant velocity, turning through a right angle every 15 minutes. Show that the snail can only return to its starting point after a whole number of hours.