MATH WE CAN DO!

1. In Flatland (a two-dimensional world on a flat plane) the post office defines the *girth* of a rectangular box as length + width. The higher the girth of a package, the more it costs to ship it. You can fit a longer rectangular box into a shorter one by fitting it diagonally; however, show that a Flatlander cannot cheat the post office by such a ruse.

2. Two Flatlanders Alex and Kolya are standing in the exact center of a square room. They turn their backs towards each other to each face an opposite wall. Each walks half the distance to their wall to end up in the final configuration shown below.

Each of the four walls of the square is completely reflective. Kolya has a laser gun that he wants to shoot at Alex, and he's allowed to reflect beams off the walls. To protect himself, Alex is allowed to put bodyguards anywhere in the room so that they'd block the path of laser. Unfortunately, these bodyguards have no length or width – they only block a laser coming directly at them. With an infinite number of guards, Alex could set up a wall between him and Kolya so that no laser could get through. Can Alex completely protect himself with only a *finite* number of bodyguards? If so, how many can he get away with?

3. Anything sent through the Flatland mail will be stolen unless it is enclosed in a padlocked case. However, Chris wants to send Dylan a birthday present. Dylan and Chris both have an exuberant number of padlocks and keys – how can these be best put to use?

4. A particularly adventurous Flatlander named Magellan has discovered that Flatland is not actually lying on an infinite, flat plane. Rather, Flatland is a huge sphere (like the surface of the earth). What can you say about planar graphs and triangles here? What if Flatland were actually on an infintely long cylinder? What about a torus (like the surface of a bagel)?

5. A polygon is cut out of paper and then folded along some line, once, and flattened. Show that the perimeter of the new polygon is not greater than that of the original.

6. Is it possible to choose 6 points on the plane and connect them by disjoint segments (i.e. segments that don't intersect on their inner points) so that each point is connected to exactly 4 others?

7. Let p be prime. Show that p mod 30 is either 1 or a prime.

8. Find the smallest number that ends with 2 with the property that moving 2 to the beginning of the number doubles it.

9. Can the sum of the digits of a perfect square equal 1970?

10. Find all three-digit numbers such that any power of that number ends with the original number (i.e. the last three digits of any power are the same).

11. We viewed symmetry groups of a regular polygon as a small part of a permutation group. Can you view modular arithmetic (addition) in this light?

12. Let's go one step more and try to view modular addition as some part of some *symmetry* group. What do the relatively prime numbers mean in this geometric interpretation?