HOMEWORK 3 FOR WINTER QUARTER

0. [Required] Algebra and Manipulation Workout:

(1) Let $a = p_1 b + r_1$, $b = p_2 r_1 + r_2$. Find x and y so that $r_2 = ax + by$.
(2) Compute the GCD of $2^5 \cdot 7 \cdot 11$ and $2^3 \cdot 11 \cdot 29$.
(3) Compute the GCD of 3 and 121.
(4) Compute the prime factorization of 693.

1. Prove the following facts, where a, b, c and n are all integers.

(1) If $a \equiv b \bmod n$ and $b \equiv c \bmod n$, then $a \equiv c \bmod n$.
(2) $a \equiv a \bmod n$
(3) If $a \equiv b \bmod n$, then $b \equiv a \bmod n$.

2. Prove the following facts, where a, b, c, d and n are all integers:

(1) If $a \equiv b \bmod n$ and $c \equiv d \bmod n$, then $a + c \equiv b + d \bmod n$.
(2) If $a \equiv b \bmod n$ and $c \equiv d \bmod n$, then $ac \equiv bd \bmod n$.

3. Show that there are no integers x and y such that $x^2 + 2 = y^2$.

4. Draw a square grid of length 10 and label the rows and columns $0, ..., 9$. In the (i, j) spot put the number $i + j$ in 'lowest form' modulo 10. Draw another grid but this time put $i \cdot j$ in the (i, j) spot. Answer the following questions about each of the tables:

(1) Do any numbers $0, ..., 9$ appear more than once in any row? Column?
(2) What is the relationship between the (i, j) spot and the (j, i) spot in the table?
(3) List all the numbers $i = 0, ..., 9$ such that there exists some other number $j = 0, ..., 9$ with $i \cdot j = 0$.
(4) List all the numbers $i = 0, ..., 9$ such that there exists some other number $j = 0, ..., 9$ with $i \cdot j = 1$. Compare with the list from the previous part.
(5) Compute GCD$(n, 10)$ for every n in the list from part (3).
(6) Compute GCD$(n, 10)$ for every n in the list from part (4).

5. (Challenge) Use the Euclidean algorithm to show that, for any integers a and b, GCD$(a, b) = ax + by$ for some integers x and y.