MATH CIRCLE WINTER QUARTER WEEK 1

0. Here's an algebra workout. Show all your work in your notebook!
 (1) Expand $(x + y)^5$. (Hint: maybe do $(x + y)^3$ and $(x + y)^2$ first.)
 (2) Simplify $\frac{n!(k-2)!}{(n-2)!k!}$.
 (3) Expand out $\frac{n^2(n+1)^2}{4}$.
 (4) Expand out $\left(\frac{n^2}{2} + \frac{n}{2}\right)^2$.
 (5) Solve for a in the equation: $ax^2 + ay - 12z = 2^n w + ak$.

Throughout the homework you are allowed to use the following fact without proof:
Given any triangle $\triangle ABC$ in the plane, the following inequalities hold:

$$AB \leq AC + BC, \quad AC \leq AB + BC, \quad BC \leq AB + AC.$$

1. Prove that the length of any side of a triangle is not more than half of its perimeter.

2. The distance from St. Petersburg to Moscow is 660 km. From St. Petersburg to the town of Likovo it is 310 km, from Likovo to Kiln it is 200 km, and from Klin to Moscow is 150 km. How far is it from Likovo to Moscow?

FIGURE 1. Map of Russia

3. A woodsman's hut is in the interior of a peninsula which has the form of an acute angle (i.e. less than 90 degrees). The woodsman must leave his hut, walk to one shore of the peninsula, then to the other shore then return home. How should he choose the shortest path?
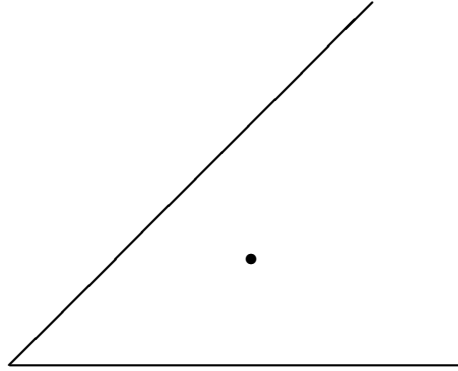
4. A fly sits on the outside surface of a cylindrical drinking glass. It must crawl to another point, situated on the inside surface of the drinking glass. Find the shortest path possible (neglecting the thickness of the glass).

5. (Challenge) Do problem (3) but this time assume that the angle is obtuse, i.e. more than 90 degrees.