MATH CIRCLE HOMEWORK 9

Announcement: The last class this calendar year is December 5; there will be a potluck holiday party after class starting at 5 pm coordinated by Lucy Kearsley (Kearsleyranch@hotmail.com). The UW math circle will resume on January 9th. There is no sign-up or registration required, but please let us know if you are not coming back.

1. Show that, for any integer $n \geq 4$, $2^n < n!$.

2. Your friend is thinking of a number between 1 and $2^n - 1$. Show that you can always figure it out in n guesses, as long as your friend tells you if you guessed too high or too low after each guess.

3. You are given a balance and a mystery object of weight at most 2^n lies on one side. Determine the minimum number of weights that you need so that you can counterbalance any such mystery object. (Assume all weights are whole numbers, and you may only place your weights on the opposite side of the mystery weight.)

4. The Fibonacci numbers are a very famous sequence of numbers defined recursively as follows:

$$F_0 = 0, \;\; F_1 = 1 \qquad F_n = F_{n-1} + F_{n-2}.$$

So the first few are:

$$0, 1, 1, 2, 3, 5, 8, 13, ...$$

Show that infinitely many Fibonacci numbers are divisible by 4.