MATH CIRCLE HOMEWORK WEEK 4

(1) You can only buy hotdog buns in packages of 4, 6, or 7. By buying different combinations of these packages, what are the only numbers of hotdog buns that you are **not** able to buy?

(2) Is the number $2^{512} - 1$ prime?

(3) Let a denote the sum of three consecutive natural numbers, and let b denote the sum of the next three consecutive natural numbers. Is it possible for the product ab to be divisible by 111111111?

(4) Prove that a number written using one 1, two 2's, three 3's, ..., and nine 9's cannot be a perfect square.

(5) If every boy in a class buys a muffin and every girl buys a sandwich, they will spend one dollar less than if every boy buys a sandwich and every girl buys a muffin. The muffins and sandwiches both cost a whole number of dollars. We know that the number of boys in the class is greater than the number of girls. Find the difference. Also, which costs more: muffins or sandwiches?

(6) (∗) Find all **integers** a and b that satisfy the equation
$$a(a + 1) = b(b + 2).$$