MATH CIRCLE HOMEWORK WEEK 3

1. Given 8 different positive whole numbers, none of which are greater than 15, show that at least three pairs of them have the same positive difference.

2. Fifty-one points are scattered inside a square with a side of length 1 meter. Prove that some set of three of these points can be covered by a square with side length 20 centimeters.

3. The alphabet of a certain language contains 22 consonants and 11 vowels. Any string of these letters is a word in this language as long as no two consonants are together and no letter is used twice. The alphabet is divided into 6 groups (all non-empty, but not necessarily of the same size). Prove that the letters in some group form a word in the language.

4. The number 458 is written on a blackboard. It is allowed either to double the number or erase the last digit. Is it possible to obtain the number 14 using these operations? If not, why not? If so, then how?

5. 175 Humpties cost more than 126 Dumpties. Is it possible to buy three Humpties and one Dumpty for one dollar? If not, why not? If so, how?