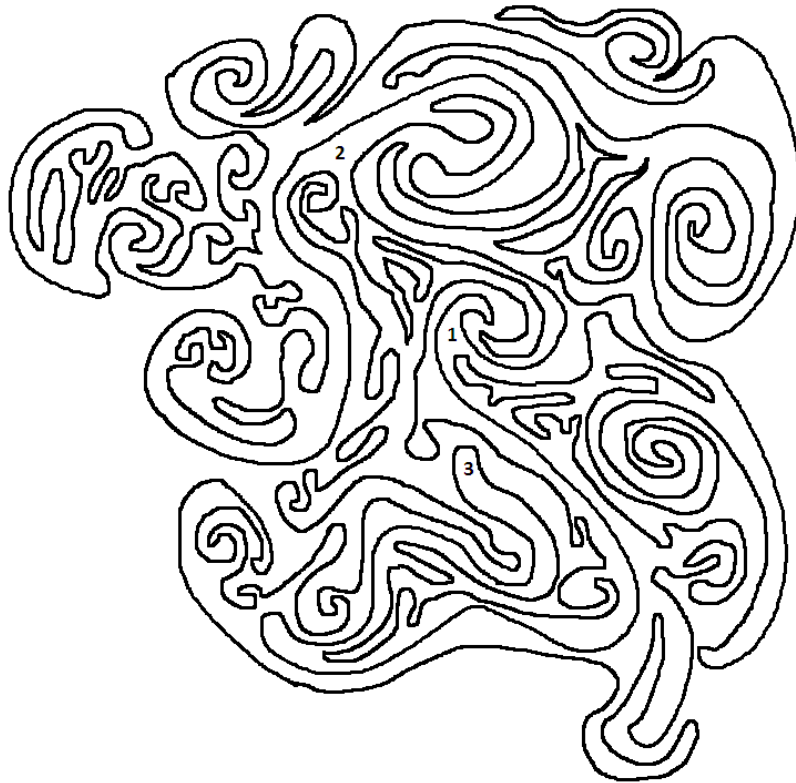# UW MATH CIRCLE
HOMEWORK 2 - PARITY AND DIVISIBILITY

(1) A 17-digit number is chosen, and its digits are reversed, forming a new number. These two numbers are added together. Show that their sum contains at least one even digit.

(2) A ladybug home is pictured below. For each ladybug 1, 2, and 3, determine whether or not the ladybug is inside or outside the house.

(3) (a) Can you arrange the digits 1 to 5 in a line from left to right, in such a way that the number formed by the first digit is divisible by 1, the number formed by the first two digits is divisible by 2, and so forth, so that the number formed be all five digits is divisible by 5? If it's not possible, make sure you can explain why not.

(b) Repeat part (a) using the digits 1 to 6, and going up to a six-digit number.

(4) Make sure you can **prove** everything you use for this problem!
   (a) What is the last digit of 2^{20100}? What about 2^{20101}?
   (b) What is the last digit of 3^{20100}? What about 3^{20101}?

(5) Suppose there are 101 people standing in a circle. Label the people clockwise from Person 1 to Person 101.

   Person 101 starts with a ball. On the first turn, she passes the ball one person to her left (so clockwise to Person 1). On the second turn, the ball then gets passed twice to the left. On the third turn, whoever has the ball passes it three people to the left, and so on. Who has the ball after the 100th turn?