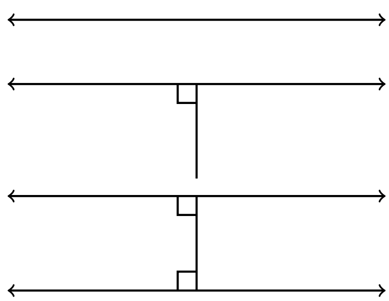
UW Math Circle

Week 6

Adapted from worksheets by Frank Sottile and UCLA Math Circle

Before you start the problems, make your spherical soccer ball and hyperbolic soccer ball using the instructions and materials provided.

1. Recall that in normal geometry, we can draw parallel lines like this:



First, draw a straight line.

Draw a segment perpendicular to the line.

Draw a straight line perpendicular to the segment.

Try doing this on your spherical soccer ball (pencil suggested). Did you get what you expected?

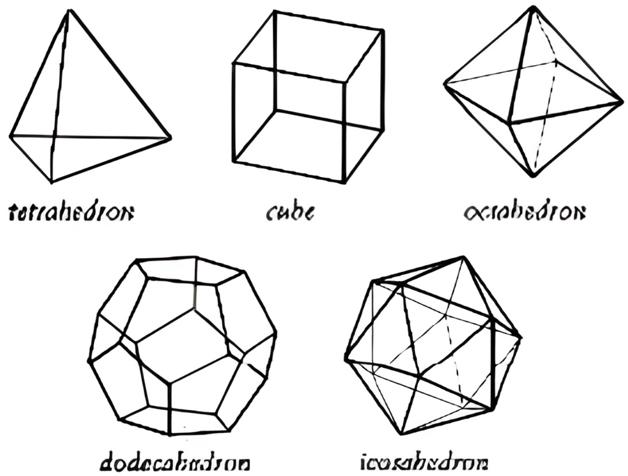
2. Recall that in normal geometry, given a straight line and a point, there is exactly one line that goes through the point and is parallel to the first line. Show that on your hyperbolic soccer ball, there are actually multiple such lines!

3. Recall that in normal geometry, the sum of angles in a triangle is 180 degrees. Draw a triangle (a shape whose sides are 3 straight lines) on both your spherical soccer ball and hyperbolic soccer ball. Now measure the angles. What do you see?

4. The *angular defect* of a vertex is $360°$ minus all of the angles that make up the vertex. For example, because 3 squares with $90°$ angles make up the vertex of a cube, the angular defect of each vertex in a cube is $360° - 90° - 90° - 90° = 90°$. What are the the angular defects of the vertices in your spherical and hyperbolic soccer balls?

5. Go back to your triangles from question 3, and count the number of vertices inside each triangle. Can you find a relationship between the angles of the triangle, the angular defect of each vertex, and the number of interior vertices? (It might help to draw more triangles, or compare triangles with your group!)

Last week, we talked about the following 3D shapes:



We counted the number of vertices, edges, and faces, and found the following formula:

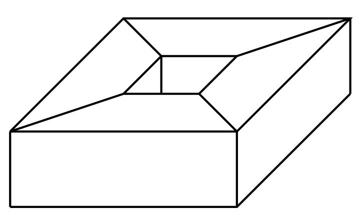
$$\text{vertices} - \text{edges} + \text{faces} = 2.$$

(If you weren't here last week, give this a quick try to convince yourself it's true!)

6. (a) A 3D shape is *closed* if every edge has a face on both sides of the edge. Are the five shapes above closed? Is your spherical soccer ball closed? Is your hyperbolic soccer ball closed?

   (b) The *total angular defect* of a closed shape is the sum of the angular defects of every vertex in the shape. Compute the total angular defects for some of your favorite closed shapes. Do you see a pattern?

7. Is the following shape closed? What is vertices − edges + faces for this shape? What is the total angular defect? (It has a hole in the middle, like a square donut.)



8. Can you think of how to draw a shape with more holes? Calculate vertices−edges+faces and total angular defect for shapes with more holes. Do you see a pattern?