

# UW Math Circle

## Week 10

1. There are 22 Math Circlers in the class. Is it true that we can find four of them who were born on the same day of the week?
2. Prove that out of any 11 integers, you can always find two whose difference is divisible by 10.
3. The Brave Knight Cosmo rolls out dough into a 4-foot-by-4-foot square, then leaves to buy some sugar at the store. When he comes back, he sees that 33 flies have landed on the dough and are feeding on it! Angry, Cosmo reaches for his 1-foot-by-1-foot square baking pan. Show that he can bring the baking pan down on the dough to kill at least 3 flies with one blow.
4. What is the greatest number of rooks you can place on an  $8 \times 8$  chessboard so that no two of them attack each other?

5. What is the greatest number of bishops you can place on an  $8 \times 8$  chessboard so that no two of them attack each other?

6. What is the greatest number of queens you can place on an  $8 \times 8$  chessboard so that no two of them attack each other?

7. Can you fill a  $3 \times 3$  table with the numbers 0, 1, and 2 in such a way that the sums along the rows, columns, and diagonals are all different? (For example, in the table below, there are several sum that are 3 and several that are 4, so that configuration doesn't work.)

			<b>/4</b>
<b>1</b>	<b>0</b>	<b>2</b>	<b>3</b>
<b>2</b>	<b>2</b>	<b>1</b>	<b>5</b>
<b>0</b>	<b>1</b>	<b>1</b>	<b>2</b>
<b>3</b>	<b>3</b>	<b>4</b>	<b>\4</b>

8. Five points are marked on the surface of a sphere. Prove that we can cut the sphere into two hemispheres such that at least four points belong to the same hemisphere. Note: if a point lies exactly on our cut, it belongs to both hemispheres.

9. Any two people at a dinner party are either friends or enemies. How many people must be at the party to guarantee that some group of three are all friends or all enemies?
10. 64 whole numbers are written on the squares of a chessboard. The numbers in each row sum to an even number, as do the numbers in each column. Show that the sum of all the numbers on black squares is also even.
11. The UW Dance Club is holding auditions for a new ensemble. Each dancer knows at least one of the following dances: Argentine Tango, Bachata, Cha-cha-cha, Discofox, East Coast Swing, and Foxtrot. The ensemble can be any (positive) size, but there must be an *even number* of members who know each dance, so everyone has a partner when they rehearse.
- How many dancers must audition to guarantee that a suitable ensemble can be formed?