Week 7

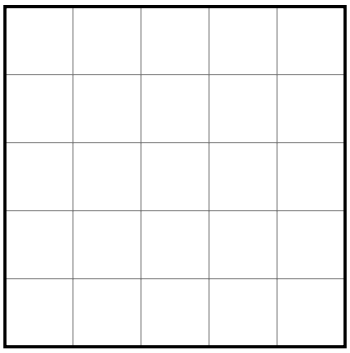
Question 1. I have 10 black socks, 6 grey socks and 8 brown socks. If I grab socks from my sock drawer without looking, what's the smallest number of socks I need to grab to guarantee that I've got at least one matching pair?

Question 2. Choose 11 numbers from $1, 2, \ldots, 20$. Is it possible to choose them so that every pair of numbers has a common factor (besides 1)? Explain why.

Question 3. Can you find 10 integers so that no subset of those integers has a sum that's divisible by 10? Explain why.

Question 4. Can you find a number where all the digits are 1 and the number is divisible by 2023? Explain why.

Question 5. Can you arrange the 26 letters of the alphabet without having six of them in either alphabetical order or reverse alphabetical order?



Question 6. My garden is a 5×5 metre square, and there are 26 snails in the garden. Explain why there must always be two snails within $\sqrt{2}$ metres of each other, no matter where the snails travel.

Question 7. I'm playing a game of go on a 5×5 go board. In go, you place stones on the intersections of the grid lines. Can I place 5 stones so that no pair of stones has a midpoint that's also an intersection of grid lines? Explain why.

Question 8. Explain why every possible arrangement of 11 rooks on a 5×5 chess board includes three rooks that are all in different rows and different columns.

Question 9. You're at a party with 21 people present in total. Every pair of people is either friends or enemies. Explain why there must be two people present who have the same number of friends as each other.

Question 10. There are 6 people at the party now (it's getting late and most people have gone home). Explain why there must be either three people who are all friends, or three people who are all enemies (or both).

What if there were 17 people present, and each pair of people is either friends, enemies or strangers — must there be a group of three people who all have the same kind of relationship with each other?

What if there's a 4th type of relationship — how many people need to be present to guarantee the existence of a group of three people who all have the same relationship?

Question 11. Can you completely colour the 2D plane with two colours, so that no two points of the same colour are a distance exactly 1 inch apart?

Here's an example colouring that doesn't work:

What if you use three colours? Or four? What's the smallest number of colours you can use to make this work?

Question 12. *(Extra problem)* Can you generalise any of the problems on this worksheet?