Week 6 auction!

Question 1. Take a positive integer and repeatedly take the sums of the squares of its digits. If this terminates at 1, call this number a decaying number. Find a decaying number less than 10,000 that takes as long as possible to terminate. For example, 10 is a decaying number because $10 \to 1^2 + 0^2 = 1$, and it takes 1 step to reach 1.

Question 2. Find the longest word that begins with some letters that all have holes in them when written in upper case, then the rest of the letters all have no holes. For example, DOG is such a word, because the first 2 letters (DO) both have holes, but the rest of the letters (just G) don't. The word must have at least one of each type of letter.

Note: We'll use `https://scrabble.hasbro.com/en-us/tools#dictionary` to decide what counts as a real word!

Question 3. Find a number n that's as large as possible so that at least a quarter of the numbers $1, \ldots, n$ are divisors of n.

For example, 5 has two divisors (they're 1 and 5), which is more than a quarter of the numbers $1, 2, 3, 4, 5$ (because $5 \div 4 = 1.25$).

Question 4. Find a list of as many different prime numbers as possible so that the average of each pair of numbers in the list is also a prime number.

For example, $3, 7, 11$ is ALMOST a valid answer: the average of 3 and 7 is prime, and the average of 3 and 11 is prime, but the average of 7 and 11 is NOT prime, so this list is invalid.

Question 5. Find an arrangement of 10 rooks on a 10×10 chess board so that the distance between the closest pair of rooks is as large as possible. (The "distance" between two squares is the number of steps you need to take from one to the other, travelling left, right, up and down.)

For example, if you put all the rooks in the first row, the minimum distance between a pair of rooks would be 1.
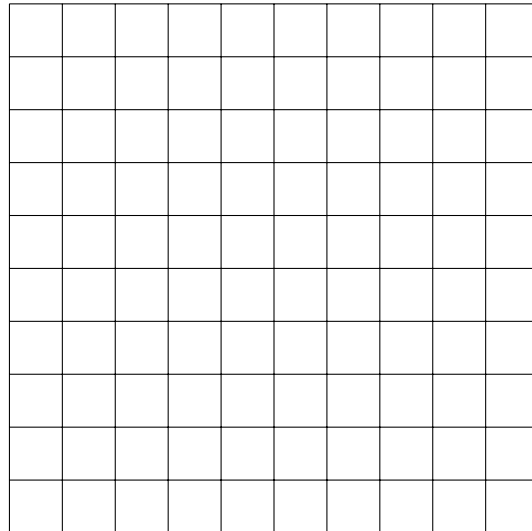
Question 6. Find the longest list of CONSECUTIVE numbers so that none of the numbers in your list are prime numbers.

For example, $8, 9$ is a valid list of length 2, because $8 = 2 \times 4$ and $9 = 3 \times 3$, and 8 and 9 are consecutive.