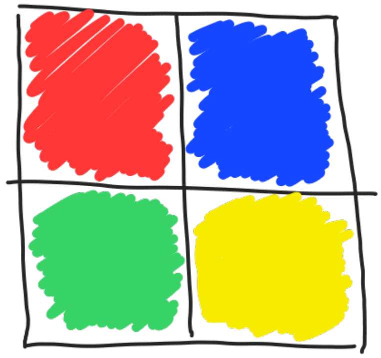
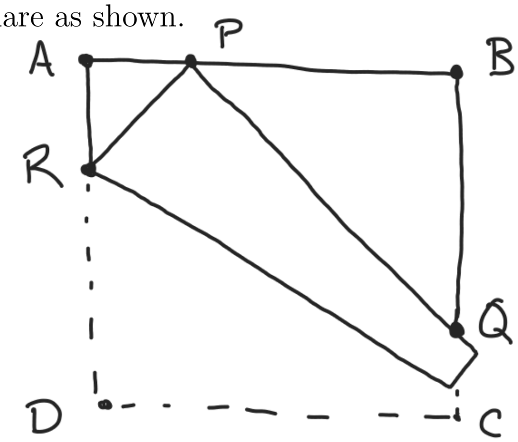
UW Math Circle

1. Color a 2 by 2 square piece of paper on both sides as shown.



   Fold the paper into a 1 by 1 square with each "layer" having only a single color. What are the possible orders for the colors to be in? What orders are impossible? Does your answer change if only one side is colored?

2. Fold a square piece of paper so that the lower left corner rests on the top horizontal line and label your square as shown.



   What is the length of the line connecting A to P? What is the length of the line connecting B to Q? Make a table comparing values as P moves along the line connecting A and B.

3. For each of the following questions, your points can be any points on a piece of paper and your lines can be any lines.

   i. Given two points p_1 and p_2, fold a line connecting them. How many ways can this be done?

   ii. Given two points p_1 and p_2, fold p_1 onto p_2. How many ways can this be done?

   iii. Given two lines l_1 and l_2, fold line l_1 onto l_2. How many ways can this be done?

   iv. Given a point p_1 and a line l_1, make a fold perpendicular to l_1 passing through the point p_1. How many ways can this be done?

   v. Given two points p_1 and p_2 and a line l_1, make a fold that places p_1 onto l_1 and passes through the point p_2.

   vi. Given two points p_1 and p_2 and two lines l_1 and l_2, make a fold that places p_1 onto line l_1 and places p_2 onto line l_2.

4. We call the six operations above "Huzita's Origami Axioms"! These operations are very close to the constructions you are able to do with a compass and straight edge. For example, if you had two points p_1 and p_2, you could use a straight edge to draw a line between them. This is very similar to the first of Huzita's axioms.

   A straight edge allows us to draw a straight line (of an unknown length) and a compass allows us to draw a circle (of an unknown radius) centered at a given point. Which Huzita's axioms would you be able to "replicate" in this manner with a straight edge and compass? Which would you not be able to replicate?

5. Take a square piece of paper. Let the bottom edge be l_1 and take p_1 to be a point in the middle and close to l_1. Then choose p_2 to be anywhere on the left or right edge of the square and perform Huzita's fifth axiom. Then choose a different p_2. Repeat this 8 or 9 times. What do you see?

6. Repeat the exercise above with Huzita's sixth axiom. What do you notice?

Bonus Questions!

1. Paper folding can solve many problems that are not solvable with a straight edge and compass. Try out the construction below for yourself and see if you can prove why it works!

   Trisecting an Angle

   (a) Let the angle you want to trisect originate from the lower left corner. Call this angle A. Make two parallel, equidistant horizontal creases at the bottom.

   (b) Then perform the sixth axiom. Fold p_1 onto l_1 and p_2 onto l_2.

   (c) With this folded, refold crease l_1, now in its new position, and extend it all the way up. This new crease, l_3, is the crease we want. Unfold step 2 and extend crease l_3 to the lower left corner (it should hit it!). The crease l_3 will mark the angle (2/3)A.

2. Suppose that we start with four points (say, p_1=(0,0), p_2=(1,0), p_3=(1,1), and p_4=(0,1) which correspond to the four corners of our square paper) and the lines l_1 = line between p_1 and p_2, l_2 = line between p_2 and p_3, l_3 = line between p_3 and p_4, and l_4 = line between p_4 and p_1. Can we find a smaller list of basic origmai operations? In other words, are any of Huzita's axioms redundant?