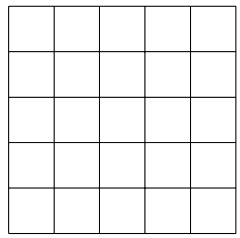
Week 2

Question 1. Wario has a 5×5 block of cheese, made up of small 1×1 squares.



Wario wants to go through the block of cheese, eating squares of cheese as he goes, always moving from one piece to an adjacent one (horizontally or vertically, not diagonally). If Wario starts at the top left corner, can he find a path through the 5×5 block that goes through every square, without returning to the same square twice?

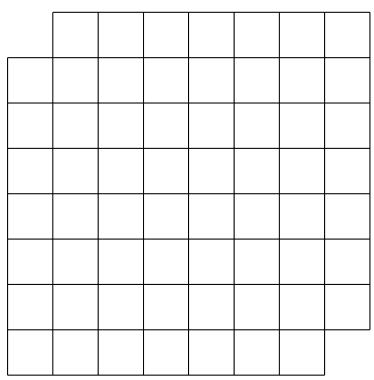
What if he starts at the square just below the top left one?

Which squares can Wario start at? Why?

Question 2. Whomp has a box full of dominoes (1×2 blocks). Can Whomp cover an 8×8 chessboard with dominoes, so that every square is covered, no dominoes overlap, and no dominoes stick out over the sides of the board?

What about a 9×9 chessboard?

Or how about this chessboard, with two corners missing?



Try drawing your own chessboard shape, and then figure out if Whomp can cover it with dominoes.

Question 3. How many different ways could Whomp cover a 2×4 chessboard with dominoes?

How about a 2×5 chessboard? Or a 1×6 chessboard, or a 3×17 chessboard?

In the table below, fill in the number of ways to cover an $m \times n$ chessboard. Try to fill as many spaces as you can, but don't worry if some are too hard.

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Do you spot any patterns? Can you explain why they happen?

Question 4. Birdo has a bunch of chess pieces. How many ways can she arrange some rooks on a 3×3 chessboard in such a way that no two rooks are in the same row or column as each other? (In other words, arrange the rooks so that none of them could attack each other, if this were a real chess game.)

What about a 2×2 chessboard? A 4×4 chessboard? An $n \times n$ chessboard?

Question 5. Bowser has 3 toasters and 3 power sockets. How many different ways can he connect his toasters to the power sockets? What if he had 2 toasters and 2 power sockets instead? Or 4 of each, or n of each?

Here's an illustration of all possible toaster/socket connections: