

UW Math Circle

Faron and Snoopy are playing a game where they alternate saying positive integers and a player loses by saying 1. However, they aren't allowed to say a number which can be written as a sum involving previously said numbers. For example, if 4 and 7 have been named, then $15 = 4 + 4 + 7$ and $12 = 4 + 4 + 4$ are not valid moves but 13 is a valid move. Here is an example of a game where Faron plays first and Snoopy plays second:

Move	New Available Moves
Start of Game	1,2,3,4,5,6,7,8,9,10,...
F: 4	1,2,3,5,6,7,9,10,11,13,...
S: 7	1,2,3,5,6,9,10,13,17
F: 9	1,2,3,5,6,10
S: 3	1,2,5
F: 5	1,2
S: 2	1

Snoopy wins the game since Faron is forced to say 1.

Find a friend and play a few games. Try to answer some the following questions:

1. If Faron plays 4 and Snoopy plays 3 then the available moves are 1, 2, 5.
Who wins and how?
2. If Faron plays 4 and Snoopy plays 5 then the available moves are 1, 2, 3, 6, 7, 11.
Who wins and how?
3. If Faron plays 4 and Snoopy plays 7 then the available moves are 1, 2, 3, 5, 6, 9, 10, 13, 17.
Who wins and how?
4. If Faron plays 4 and Snoopy plays 6 then the available moves are 1, 2, 3, 5, 7, 9, 11, 13, ...
Who wins and how?
5. Is 2 or 3 or 4 or 6 a good first move?
6. Could the game go on forever?

An *ender* is a move which is eliminated by every other move. For example, after 4,7 have been played, the remaining moves are 1,2,3,5,6,9,10,13,17 and 17 is an ender.

7. Show that if an ender exists, it must be the largest available move.
8. Use the strategy stealing idea from Chomp to show that if an ender larger than 1 exists, the player next to move can force a win.

Using problem 6 and some number theory, it is possible to show the following useful fact:

- If the first two moves played share no common factors (other than 1) then Faron can force a win (unless the first two moves are 2 and 3).

For example, if Faron plays 4 and Snoopy plays *any* odd number then Faron can force a win. Play a few more games and try to answer some of the following questions:

9. Is 5 a good first move for Faron?

10. Is 15 a good first move for Faron?